Solution to Electrical Impedence Equation

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Objetive

Using the transmutation operator describe in [1, 2], we introduce an quaternionic operator operator **T**, such that transforms a monogenic functions into solutions of the electric impedance equations, due to in [3, 4] found a way to transform the electric impedance equation into a quaternionic differential form.

Method

Consider the equation

$$(\operatorname{div} \sigma \operatorname{grad} + q)u = 0 \text{ in } \Omega \subset \mathbb{R}^3 \tag{1}$$

where σ, q, u are complex valued functions, $\sigma \in C^2(\Omega)$ and $\sigma \neq 0$ in Ω in [3] Kravchenko gives a factorization of (1) and a way to construct solution to these equation.

Theorem 1. *[3, Theorem 159] The equation* (1) *can be factorized as*

$$(\operatorname{div} \sigma \operatorname{grad} + q) = -\sigma^{1/2}(D + M^{\vec{\sigma}})(D - M^{\vec{\sigma}})\sigma^{1/2} \tag{2}$$

The case when $q = 0$ has important applications, one is in medical problems specifically in Electrical Impedance Tomography these equation has a main role.

$$\operatorname{div}(\sigma \operatorname{grad} u) = 0. \tag{3}$$

These equation is known *electrical impedance equation*, where σ is the conductivity function and u denotes the electrical potential. In [5] shows the following process to turn (3) into $D + M^{\vec{\sigma}}$, where $\vec{\sigma} = D\sqrt{\sigma}/\sqrt{\sigma}$, for this reason we only works with conductivity functions who are separable i,e $\sigma(x) = \sigma_1(x_1)\sigma_2(x_2)\sigma_3(x_3)$.

Proposition 2. *[4, 5] Let* $\mathbf{E} = -\operatorname{grad} u$. *Then*

$$\operatorname{div}(\sigma \mathbf{E}) = 0 \tag{4}$$

is equivalent to

$$(D + M^{\vec{\sigma}})\vec{\mathcal{E}} = 0 \tag{5}$$

where $\vec{\mathcal{E}} = \sqrt{\sigma}\mathbf{E}$ *and* $\vec{\sigma} = \frac{D\sqrt{\sigma}}{\sqrt{\sigma}}$.

Results

We construct an *invertible operators* $\mathbf{T}, \widetilde{\mathbf{T}}$ that acts on complex quaternion functions such that related the well know monogenic function into solutions of $D + M^{\vec{\sigma}}$

Theorem 3. *For* $v \in C^1(\Omega, \mathbb{H}(\mathbb{C}))$,

$$\left(D + M^{D\sigma/\sigma}\right) \mathbf{T}[v] = \widetilde{\mathbf{T}}[Dv]. \tag{6}$$

Every solution of u *of* (3) *can be obtained as*

$$u = \mathcal{B}\left[\frac{\mathbf{T}[\vec{v}]}{\sqrt{\sigma}}\right] \tag{7}$$

where $\mathcal{B}$ *is an antigradient operator.*

Conclusions

With the aid of the transmutation operators we are able to construct a solutions of the electrical impedance equation, moreover since the operator $\mathbf{T}$ is invertible we can approximate every solution of (3) with this method, using a special type of polynomials.

References

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